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Experimental investigation on the hydrodynamics of falling liquid film flow by nonlinear description procedure

J.T. Zhang[a], X.F. Peng[a,*], G.P. Peterson[b]

[a]*Thermal Engineering Department, Tsinghua University, Beijing 100084, People's Republic of China*
[b]*Department of Mechanical Engineering, Texas A&M University, College Station, TX 77843, USA*

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Abstract

Extending a previous analytical investigation, the effect of wall heating on the hydrodynamics of falling liquid films was studied by calculating the fractal dimensions of reconstructed phase spaces from experimental measurements. The results illustrated that the wall heat flux has a significant influence on the hydrodynamics of falling liquid films, especially in the case of low flow rates. The causes for this effect may be attributable to density variations within the films and thermocapillarity effects acting on the free surface interface of the films, particularly for situations where wall heating is present. The combined effect of these two factors may be more apparent for thin films with a low flow rate than for thicker films with higher flow rates. The results indicated that the hydrodynamics and heat transfer of falling liquid films present a conjugate problem, especially in the case of low flow rates, and that this conjugation should be considered in any the study of heat transfer of falling liquid films. © 2000 Elsevier Science Ltd. All rights reserved.

Keywords: Falling liquid film; Hydrodynamics; Fractal dimension

1. Introduction

The flow of falling liquid films is a complicated phenomenon and involves a number of parameters and characteristics not previously attributed to this physical process. Decades of previous studies have, however, confirmed that the flow of falling liquid films is intrinsically unstable [1,2]. Typically, one finds small or fine waves on the free surface interface of films at, or near the exit of orifices or flow distributors. Experimental results have confirmed that these small waves coalesce into larger, sometimes solitary waves as the film continues to flow downwards and move farther and farther from the exit [3–6]. Previous theoretical and experimental studies have indicated that falling liquid film flow is transient and nonlinearly unstable, and has a "chaotic" nature [7–11]. Statistical methods have previously been widely used to reveal the flow characteristics of these falling liquid films [12,13]. More recently, a number of researchers [14,15] have employed nonlinear methods to describe the wave characteristics of falling liquid films. In these studies, the phase space was reconstructed from the experimental time series of the film thickness within the framework of deterministic chaos. As a result, several indexes of deterministic chaos analyses were extracted

* Corresponding author. Tel.: +86-10-6278-9751; fax: +86-10-6277-0209.

E-mail address: pengxf@linux.te.tsinghua.edu.cn (X.F. Peng).

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Nomenclature

d	fractal dimension
k	embedding dimension
m	dimension of compact manifold
q	heat flux
Re	Reynolds numbers, $Re = 4\Gamma/\mu$
x_i	time series
y_i	point of reconstructed phase space

Greek symbols

Γ	liquid mass flow rate per width
δ	distance to the nearest neighbor
$\langle\delta\rangle$	average δ
μ	dynamic viscosity
τ	delay time

from these reconstructed spaces to describe the hydrodynamics of falling liquid film flow. Both of the previous investigations primarily concerned with the hydrodynamics of the film flow and as a result, were conducted under the conditions of no wall heating.

Several other previous investigations have demonstrated that the presence of wall heating may have an important effect on the wave nature of the falling liquid film [16,17]. In many cases, the experimental investigations were conducted in such a manner that this effect was visible to the naked eye. As stated in the literature [18–20], asymptotic motion in dissipative systems takes place on sets which usually have zero volume in state space. If such a set is chaotic, it usually has a non-integer dimension (fractal dimension), which is smaller than the dimension of the state space. The chaotic nature of the falling liquid provides some indication that if determined, the fractal dimension could be used to demonstrate the effect of wall heating on the film behavior.

In the current investigation, phase spaces were reconstructed from experimental time series, and the fractal dimensions were calculated. These calculated fractal dimensions indicated that, the hydrodynamic characteristics of falling liquid films with and without wall heating are different.

2. Experimental apparatus

The experimental apparatus shown in Fig. 1 was designed and constructed. This test apparatus is similar to one previously described by the authors [21], and some of the results utilized in the previous work were also utilized herein.

The entrance section of the test apparatus consisted of a 90 mm long plexiglass cylinder, 18 mm outer diameter. The test section was fabricated from a 300 mm long, 0.5 mm thick and 18 mm outer diameter stainless steel cylinder. The surface finish and measurement uncertainty in the outer diameter were 1.6 μm and 0.05 mm, respectively. The falling liquid film was formed from a 0.5 mm width circle orifice. A 100 mm inner diameter, 150 mm deep water container at the end of the test section and the upper water tank was used to keep the water level stable.

A optical–electronic method was used to measure the thickness of falling liquid film [18]. A sketch of the measuring system is shown in Fig. 2. The basic principle is as follows. A semiconductor laser beam from laser 1 is expanded by lens 2. The beam is then focused by a cylindrical lens 3 to form a light sheet at the measurement spot, with a sheet thickness of less than 0.5 mm. In experiments, the focus point of the cylindrical lens is positioned on the dashed line 8 in Fig. 2. The light sheet is stretched in the horizontal direction to the length scale of the cylindrical lens which is much wider than the liquid film thickness, but the beam is quite thin in the film flow direction. Hence, the entire liquid film is within the sheet to ensure a better spatial resolution. When a band of light reaches the surface of a transparent object, part of the light will be reflected and part will be refracted. The reflected and refracted light will stray from the original direction if the incident light is not perpendicular to the surface. Hence, as shown in Fig. 2, a photodiode or screen can be placed directly behind the test model to indicate the light that does not pass through the object. Then, in the ideal situation, the light encountering liquid film is completely deflected from the direction of the incident

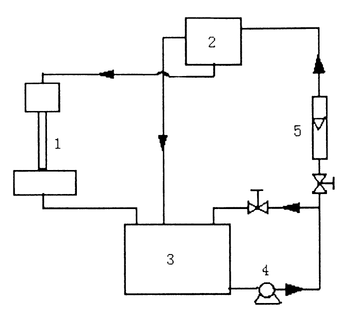

Fig. 1. Sketch of the experimental system. 1 — Entrance and test section, 2 — upper tank, 3 — lower tanker, 4 — pump, 5 — flowmeter.

light sheet and the intensity variation behind the model will be related to the change of the liquid film thickness. As a result, the intensity variation sensed by photodiode 7 will represent the changes of the film thickness.

The measuring system used a 2 mW semiconductor laser as the light resource. A 10 times expander was used to expand the laser light. The 15×15 mm cylindrical lens was used to focus the expanded laser light on the liquid film with a focal length of 40 mm. A 25 mm diameter flat convex lens was aligned with the laser light axis 90 mm behind the focal point. The focal length of the convex lens was 50 mm. The falling liquid film thickness signals were magnified with this convex lens. The screen and the photodiode were placed 600 and 270 mm behind the flat convex lens, respectively. The system was calibrated on line. During the experiment, a video camera was used to measure the average film thickness by taking pictures of transient wavy films, then using image software to determine the shift length of wave interface from the original. The practical film thickness could thus be determined for every picture and the average film thickness could then be calculated. The magnification coefficient of the video camera system was 0.50 mm/65 TV lines. To assure the measurement precision, a 450 line video camera was used with a shutter speed of 1/1000 s.

The photodiode with a 2.7×2.7 mm optical window was also utilized to measure the relative shift of the film thickness from the average value. The noise level was below 0.4 mV with pulse ascendant and descendant time intervals of less than 3.5 μs. Its response frequency band was approximately 0.1 MHz. The response sensitivity was 2500 mV/μW (1 μW variation of incident laser of wavelength 850 nm would cause a 2500 mV output). The wavelength of the semiconductor laser was about 650 nm which was within the operating range of the photodiode. The photodiode response at the semiconductor wavelength was about half that of the 850 nm wavelength. The photodiode was mounted on the base of a micrometer. After every test run, the photodiode was calibrated without the liquid film by shifting the photodiode by 0.05 mm steps from one edge of the sensor window to the opposite edge while recording the photodiode output at the starting and ending spot of every shift. The photodiode output signals are assumed to vary approximately linearly with film thickness over every 0.05 mm step. For wave amplitudes of about 0.5 mm, the measurement error caused by this assumption would be small. Smaller micrometer steps could improve the measurement precision.

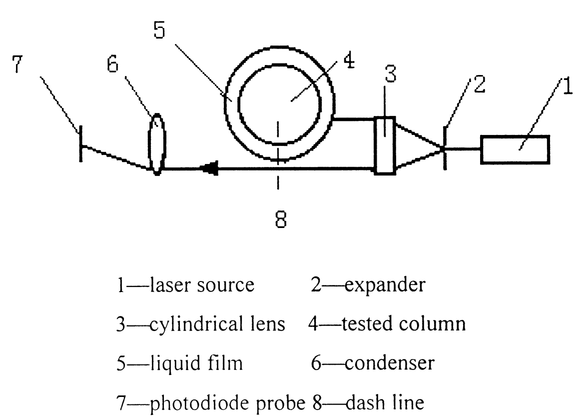

Fig. 2. Sketch of the film thickness measuring system. 1 — Laser source, 2 — expander, 3 — cylindrical lens, 4 — tested column, 5 — liquid film, 6 — condenser, 7 — photodiode, 8 — dash line.

A HP 34970A Data Acquisition/Switching Unit and a HP 34902A 16-channel multiplexer were used for the data acquisition. The system has $6_{1/2}$-digit multimeter accuracy with stability and noise rejection. The reading rate is up to 600 readings per second on a single channel. In this experiment, the integration interval was set to 0.2 PLC (equivalent to 4 ms); the corresponding resolution was $5_{1/2}$; and the attached noise error was (range of measurement) $\times$ 0.1%.

In every test run, the experimental data were collected by a PII/300 PC. Ten thousand data points were collected in each data set. The time interval between the data points was 0.015 s. The uncertainty of the sensor output was measured as below 0.4%.

3. Phase space reconstruction

The hydrodynamics of falling liquid film flow can be represented by a system characterized through the time evolution of the physical properties, such as film thickness. Basically, the hydrodynamic characteristics can be modeled by a group of partial differential equations. In essence the above system can be described by the vector variables x consisting of n independent components. The state of the system at a given time can be determined by a point x in the state space represented by R^n. The state space then has an infinite dimension for the system described by a group of nonlinear partial differential equations. The asymptotic (for large time) solutions of nonlinear equations may have complex non-periodic structures associated with their exponential divergence and sensitive dependence on initial conditions. The concept of deterministic chaos is related to this asymptotic behavior. For the open dissipative systems, such as film flow, the volume of the state space decreases to zero asymptotically. The asymptotic trajectories in state space fill out a chaotic attractor characterized by a set of non-integer dimensions. Therefore, the asymptotic orbits are within a subset of the state space with a lower dimension than that of the original.

The studies of Takens [19] and Packard et al. [20] indicated that, a phase portrait of chaotic trajectories, equivalent in some sense to that of the underlying dynamic system, could be reconstructed from time series. The commonly used reconstruction procedure is the method of delay. This method can be briefly summarized as follows. For a time series $x(t_0), x(t_1), \ldots, x(t_N)$, a phase space trajectory can be reconstructed by choosing a delay time τ and form a series of k-tuples $y_i = [x(t_i), x(t_i+\tau), \ldots, x(t_i+\tau(k-1))]$. When the embedding dimension k is chosen properly, this series, y_i, represents a trajectory that is diffeomorphic to a corresponding solution of the governing evolution equations. In this reconstruction, N should be large. The embedding dimension is a minimum dimension of an Euclidean space R^k such that there is a smooth transformation from the original state space to R^k such that it is one-to-one at every point of the attractor. Takens' theorem demands that $k \geq 2m+1$, where m is the dimension of a compact manifold, M. Takens' theorem states that a compact manifold M with finite dimension, m, containing the attractor exists although the state space is of infinite dimension.

For an infinite amount of noise-free time series, the time delay, τ, can in principle be chosen almost arbitrarily. However, the experimental time series are neither infinite nor noise-free. Therefore, the delay time should be chosen so that the coordinates of the reconstructed space are mutually independent. Fraser [22] suggested the use of the mutual information as the criterion for the choice of τ. The procedure used here was to calculate the mutual information of $x(t)$ and $x(t+\tau)$, in order to determine how dependent the values of $x(t+\tau)$ were on the values of $x(t)$. The value of τ which produces the first local minimum of the mutual information can be used for phase portraits. In this paper, the method of Fraser [22] was used to reconstructed the phase space.

4. Fractal dimension of reconstructed phase space

The nearest-neighbor method of Badii and Politi [23] was used to estimate the fractal dimension. As stated in the literature [24], this method appears to be more accurate than the more widely used correlation dimension procedure. Consider, for example, a set of N points of an experimental series that lie on the reconstructed phase space. A point, x, in this phase space can be arbitrarily selected as a reference point and a subset of n points denoted by y_i ($i = 1, 2, \ldots, n$, $n < N$) from the original set of N points can be chosen at random. The term δ is then defined as the distance to the nearest neighbor i.e., $\delta = \min\|x - y_i\|$. This calculation is repeated over many randomly chosen reference points and a average $\langle\delta\rangle$ is obtained. The process is then repeated for a sequence of n values up to $n = N-1$. It is argued that $\langle\delta\rangle \sim n^{-1/d}$ where d is the fractal dimension of the chaotic attractor. Hence, the negative, inverse slope of a $\log\langle\delta\rangle$ vs. $\log n$ plot is the fractal dimension. In order to alleviate the effect of noise in experimental time series, δ should be calculated for the 10th or 100th nearest neighbor [25]. The phase space was reconstructed by the experimental time series of this study with the above-mentioned methods. The available time series included 10,000 points. The state parameters of the experiments are listed in Table 1.

The calculated fractal dimensions indicates that the flow of falling liquid films is chaotic, regardless of whether the wall is heated or not. This is consistent with the conclusion of the previous experimental observations, where the flow of falling liquid film was considered to be turbulent at the Reynolds number greater than 250–400 [26]. Table 1 further illustrates that the fractal dimensions of the Reynolds numbers 1250 and 1690 have the maximum values; the lowest values for Reynolds numbers 2280 and 3150; and the middle value for the Reynolds number 1140. The precise relationship between the fractal dimension and the hydrodynamic characteristics of the film flow is unknown. However, it is clear that the fractal dimension varies directly with the wall heat flux for the cases of Reynolds numbers ranging from 1140 to 2280. It does appear, however, that no rule dominates the variation between the fractal dimension and the flow hydrodynamics. The calculated fractal dimensions vary irregularly with wall heat flux at the lowest Reynolds number.

Table 1
Experimental parameter and calculated fractal dimension

		(a) $Re = 1140$			
q (W/m^2)	0.0	1900	5100	8000	11,000
d	8.59	8.89	8.32	8.06	9.47
		(b) $Re = 1250$			
q (W/m^2)	0.0	1800	4900	10,500	
d	9.35	10.64	10.33	10.23	
		(c) $Re = 1690$			
q (W/m^2)	0.0	1800	4800	10,800	
d	9.11	8.94	9.44	9.45	
		(d) $Re = 2280$			
q (W/m^2)	0.0	1800	5300	10,600	17,800
d	7.31	6.84	7.19	7.57	7.47
		(e) $Re = 3150$			
q (W/m^2)	0.0	4700	11,100	14,300	18,000
d	7.55	7.68	7.65	7.55	7.55

In the cases of Reynolds numbers 1250, 1690 and 2280, the calculated fractal dimensions apparently vary with the heat flux, but remain nearly constant for the high heat flux cases. For the case where the Reynolds number is 3150, the flow hydrodynamics appear to be insensitive to the heat flux. This implies that the characteristics of falling liquid film flow are invariant with heat flux at high flow rates. As a result, it can be concluded that wall heating has an influence on the hydrodynamics of falling liquid films, and that this influence is quite strong for the cases of low flow rates. This result is consistent with the analytical conclusions found in the literature [17]. And it is also consistent with the conclusion of experimental measurements [13]. The previous investigations [13] stated that the effect of increasing the heat flux on the variation of the film thickness was stronger at low Reynolds numbers than for high Reynolds numbers. This variation makes the results insensitive to the heat flux for $Re \geq 5000$. Based upon the previous experimental results [13], it is apparent that the nondimensionalized variance of film thickness with increasing heat flux becomes narrower in the region of high flux.

The effect of wall heating on the hydrodynamics of falling films might result from two causes. First is that the effect of the buoyancy is strong in the case of low flow rates. This causes lateral motions within the film that makes the flow different from the original conditions. Because the film thickness is small in the case of low flow rates, the effect of buoyancy might also be more apparent, and it has stronger influence on the wave pattern than for high flow rates. Another cause is the role that the thermocapillarity plays on the free surface interface of the liquid film. The falling film flow is intrinsically unstable, and the free interface is wavy. The experimental results have confirmed that the temperature of the wave troughs on the free interface is higher than for the wave crests for the case of wall heating, and as a result, the thermocapillary force will draw liquid from the wave troughs to the wave crests. This process will distort the original waveform. However, the thermocapillary effect might become weaker for the thick films, so that the influence in this case is not as strong as it is for the thin films.

5. Conclusions

The calculated fractal dimension of the reconstructed phase space has confirmed that wall heating has an effect on the hydrodynamics of falling liquid films, especially in the case of low flow rate. The reasons for this can be attributed to the function of buoyancy within films and the thermocapillarity on the free interface of the films. The experiments illustrate that this influence is apparent in the case of low heat flux and but remains invariant for the case of high heat flux of Reynolds number range from 1250 to 1690. For the flow of Reynolds number 3150, the flow hydrodynamics appear to be almost insensitive to the wall heating. For the flow of Reynolds number 1140, the influence is always strong. This phenomenon implies that the flow hydrodynamics and wall heating of falling liquid films are coupled. In addition, the flow characteristics are quite complicated and cannot be described completely by the sole parameter of either mass flow rate or Reynolds number. The addition of heat has also has an effect, such as the prediction of dryout critical heat flux. The experimental results [21] indicated that thermocapillarity plays an important role in film burnout. The streamwise thermocapillarity term in the newly developed model [21] is quite effective in improving the precision of the dryout heat flux prediction. This term has a direct relationship with wall heat flux. Although the fractal dimension can exhibit the influence of wall heating on the hydrodynamics of falling liquid film flow, the deeper relationship between the fractal dimension and the hydrodynamics of falling film flow is still as yet unknown.

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